# Free Vibration Analysis of Perforated Plate Submerged in Fluid

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An analytical method to estimate the coupled frequencies of the circular plate submerged in fluid is developed using the finite Fourier-Bessel series expansion and Rayleigh-Ritz method. To verify the validity of the analytical method developed, finite element method is used and the frequency comparisons between them are found to be in good agreement. For the perforated plate submerged in fluid, it is almost impossible to develop a finite element model due to the necessity of the fine meshing of the plate and the fluid at the same time. This necessitates the use of solid plate with equivalent material properties. Unfortunately the effective elastic constants suggested by the ASME code are found to be not valid for the modal analysis. Therefore in this study the equivalent material properties of perforated plate are suggested by performing several finite element analyses with respect to the ligament efficiencies.

**Key Words :** Perforated Circular Plate, Fluid-Structure Interaction, Equivalent Material Property, Ligament Efficiency

## 1. Introduction

The analysis of multiholed plate has attracted the attention of many engineers and designers due to the widespread use of tubesheet heat exchangers and other similar equipments. The stress analysis of a plate perforated with a large number of holes, by finite element method for instant, was a very costly and time-consuming technique which solves only one particular problem. But it is possible to model the perforated plate and to analyze it and it is no more time-consuming theses days due to the rapid development of the computer and software. However, if a perforated plate is submerged in fluid it is almost impossible to model and analyze it as is and the fluid at the same time, which is needed to investigate the effect of the fluid-structure interaction. The simplest way to avoid time-consuming and costly analysis of perforated plate submerged in fluid is to replace the perforated plate by an equivalent solid one considering weakening effect of holes.

For a long time a general method was required which could replace the real drilled plate by an equivalent undrilled one of the same dimensions for which the classical solid plate theory, in the elastic range, would be applicable. This equivalent plate must have the appropriate elastic constants so as to show the same behavior as the real one when subjected to the same loading.

These effective elastic constants must be correctly evaluated, especially in fixed tubesheet heat exchangers. If they are too low, the stresses at the junction with shell and head will be lower than

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those in realty. If they are too high, the stress at the center of the plate, which may be the maximum, is too low. Thus the proper design can only be obtained if the analysis is performed using the best possible estimation of the effective elastic constants.

Many authors have proposed experimental or theoretical method to solve this problem. Slot and O'Donnell (1971) determined the effective elastic constants for the thick perforated plates by equating strains in the equivalent solid material to the average strains in the perforated material. O'Donnell (1973) also presented those of thin perforated plates. These results are implemented in Article A-8000 of Appendix A to the ASME code Section III (ASME, 2004), which contains a method of analysis for flat perforated plates when subjected to directly applied loads or loadings resulting from structural interaction with adjacent members. This method applies to perforated plates which satisfy the conditions of (a) through (e) below.

(a) The holes are in an array of equilateral triangles.

(b) The holes are circular.

(c) There are 19 or more holes.

(d) The ligament efficiency is greater than 5%.

(e) The plate is thicker than twice the hole pitch. If only in-plane loads or thermal skin stress are considered this limitation does not apply.

As mentioned previously, the equivalent material properties presented in the ASME code is originally determined based on the deflection due to the loading, which considers only the first mode from the modal characteristic point of view. Therefore it is necessary to redefine the equivalent material properties of a perforated plate for the modal analysis.

This study deals with the free vibration characteristics of circular perforated plate submerged in fluid, which is assumed to be incompressible, irrotational and frictionless and is bounded in the radial direction. The natural frequencies of the fluid-coupled system are obtained by theoretical calculations and verified by three dimensional finite element analyses. The effect of holes on the modal characteristics is investigated and also new equivalent elastic constants are proposed, which can be used for the modal analysis of the perforated plate.

## 2. Theoretical Development

### 2.1 Formulation

Considering a single circular solid plate submerged in a fluid-filled rigid cylinder as shown in Fig. 1, where $R$, $h$, $d_1$ and $d_2$ represent the radius and thickness of the plate, and height of upper and lower fluid respectively, the following assumptions are made for the theoretical development:

(a) the fluid motion is so small that it is considered to be linear,

(b) the fluid is incompressible, inviscid and irrotational,

(c) the material of plate is linearly elastic, homogeneous and isotropic.

The equation of motion for transverse displacement, $w$, of this perforated plate in contact with fluid is:

$$D^*\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\rho^* h\frac{\partial^2 w}{\partial t^2}=p^* \quad (1)$$

where $D^*=E^* h^3/12\ (1-\mu^{*2})$ is the flexural rigidity of the plate; $\rho^*$, $\mu^*$, $p^*$ and $E^*$ are density, Poisson's ratio, hydrodynamic pressure on the perforated plate and Young's modulus of the perforated plate, respectively.

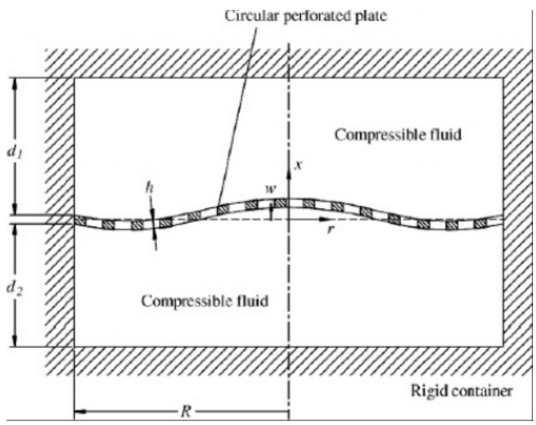



**Fig. 1** Plate submerged in fluid

The solution of Eq. (1) takes the following form of combinations for plate deformation with respect to polar coordinates $(r, \theta)$:

$$w(r,\theta,t)=\cos(n\theta)\sum_{m=1}^{M} q_m W_{nm}(r)\exp(\mathrm{i}\omega t) \quad (2)$$

where $q_m$ is unknown coefficient and $n$ and $m$ are the numbers of the nodal diameters and circles of the plate, respectively. For the plate with clamped boundary condition, the displacement along the edge of the plate, $r=R$, must be zero and therefore dynamic displacement of Eq. (2) will be reduced to:

$$W_{nm}(r)=\mathrm{J}_n(\lambda_{nm}r)-\mathrm{J}_n(\lambda_{nm}R)\frac{\mathrm{I}_n(\lambda_{nm}r)}{\mathrm{I}_n(\lambda_{nm}R)} \quad (3)$$

where $\lambda_{nm}$ is the frequency parameter for the plate in air, which is also determined by the boundary conditions and is related to the circular frequency of the plate in air $\omega$. $\mathrm{J}_n$ and $\mathrm{I}_n$ are the Bessel function and the modified Bessel function of the first kind, respectively. For the fixed boundary condition, the eigenvalues $\lambda_{nm}$ for the plate in a vacuum can be obtained from the zero slope and zero moment boundary conditions as follows (Bauer, 1995):

$$\frac{\mathrm{J}'_n(\lambda_{nm}R)}{\mathrm{J}_n(\lambda_{nm}R)}=\frac{\mathrm{I}'_n(\lambda_{nm}R)}{\mathrm{I}_n(\lambda_{nm}R)} \quad (4)$$

### 2.2 Velocity potential

The fluid region contained in cylindrical rigid vessel is bisected into two parts, an upper fluid and a lower fluid by the circular plate. The three dimensional oscillatory fluid flow in the cylindrical coordinates can be described by the velocity potential. The facing side of the circular plates is contacted with inviscid and incompressible fluid. The fluid movement due to vibration of the plate is described by the spatial velocity potential that satisfies the Laplace equation:

$$\nabla^2\Phi(r,\theta,x,t)=\frac{\partial^2\Phi(r,\theta,x,t)}{\partial t^2}/c^2 \quad (5)$$

where $c$ is the speed of the sound in the fluid. It is possible to separate the function $\Phi$ with respect to $r$ by observing that in the radial direction the vessel which supports the edges of the plate are assumed to be rigid, as in the case of the completely contact circular plate. Thus:

$$\Phi(r,\theta,x,t)=\mathrm{i}\omega\phi_j(r,\theta,x)\exp(\mathrm{i}\omega t),\quad j=1,2 \quad (6)$$

where the upper fluid is referred to with a subscript "1" while the lower fluid is denoted by a subscript "2".

For the bounded fluid, the boundary condition along the cylindrical vessel wall assures the zero fluid velocity in the radial direction given by:

$$\left.\frac{\partial\phi_j}{\partial r}\right|_{r=R}=0 \quad (7)$$

When it is assumed that all the vessel walls are rigid and the plate thickness is negligible comparing with the vessel height, the velocity potential must satisfy the followings:

$$\frac{\partial\phi_1(r,\theta,d_1)}{\partial x}=0 \text{ for the upper fluid} \quad (8a)$$

$$\frac{\partial\phi_2(r,\theta,-d_2)}{\partial x}=0 \text{ for the lower fluid} \quad (8b)$$

### 2.3 Method of solution

Since the plate thickness is neglected, the compatibility conditions at the fluid interface with the solid plate yield:

$$\left.\frac{\partial\phi_1(r,\theta,x)}{\partial x}\right|_{x=0}=w(r,\theta) \quad (9a)$$

$$\left.\frac{\partial\phi_2(r,\theta,x)}{\partial x}\right|_{x=0}=w(r,\theta) \quad (9b)$$

For the perforated plate equation (9) is not true because upper and lower fluid can move through the hole, generating very complex velocity potential.

When the gravity is neglected, it is useful to introduce the Rayleigh quotient in order to calculate the coupled natural frequencies of the circular plate submerged in the ideal fluid.

$$\omega^2=\frac{V_d}{T_d+T_F} \quad (10)$$

where $V_d$ is the potential energies of the plate and $T_d$ and $T_F$ are the reference kinetic energies of the plate and the fluid, respectively. In order to perform numerical calculations for each fixed $n$ value, a sufficiently large finite $M$ number of terms must be considered in all the previous sums of the expanding term, $m$. For this purpose, a vector $q$ of the unknown parameters is introduced as :

$$\mathbf{q}=\{q_1 \ q_2 \ q_3 \ \cdots\cdots \ q_M\}^T \tag{11}$$

Now, it is necessary to know the reference kinetic energies of the plate and containing fluids in order to calculate the coupled natural frequencies of the circular plate in contact with fluids. Using the hypothesis of irrotational movement of the fluid, the reference kinetic energy of the fluids can be evaluated from its boundary motion.

$$T_F=\frac{1}{2}\rho_o\int_0^{2\pi}\int_0^R w\phi_2(r,-d_1)\,r\,dr\,d\theta + \frac{1}{2}\rho_o\int_0^{2\pi}\int_0^R w\phi_1(r,d_2)\,r\,dr\,d\theta \tag{12}$$

The reference kinetic energy of the circular plate is presented :

$$T_d=\frac{1}{2}\rho^* t\int_0^{2\pi}\int_0^R rW_{nm}^2 dr\,d\theta \tag{13}$$

The maximum potential energy of the plate can be approximated by

$$V_d\approx\frac{1}{2}D^*\lambda_{nm}^4\int_0^{2\pi}\int_0^R W_{nm}^2 r\,dr\,d\theta \tag{14}$$

The correspondence between the reference total kinetic energy of each mode multiplied by its square circular frequency and the maximum potential energy of the same node are used. In order to find natural frequencies and mode shapes of the plate in contact with fluid, the Rayleigh quotient for the plate vibration coupled with ideal fluid is used. Minimizing Rayleigh quotient $V_d/(T_d+T_F)$ with respect to the unknown parameters $q_m$, the non-dimensional Galerkin equation can be obtained and an eigenvalue problem and the natural frequencies $\omega$ can be calculated.

## 3. Analysis

### 3.1 Theoretical analysis

On the basis of the preceding analysis, the non-dimensional Galerkin equation is numerically solved using MathCAD in order to find the natural frequencies of circular plate coupled with fluid. In order to check the validity and accuracy of the results from the theoretical study, finite element analyses are also performed and frequency comparisons between them are carried out for the fluid-coupled system.

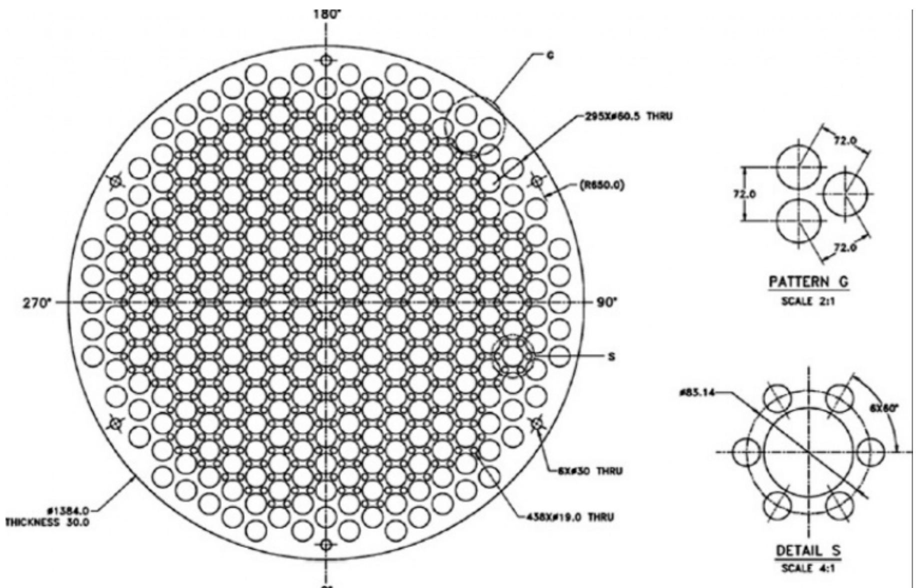



**Fig. 2** Perforated plate

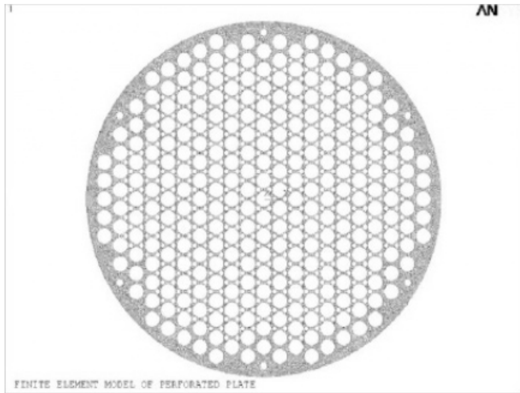
The model of the fluid-coupled structure simulates the lower control element guide plate of an integral reactor, which is a circular perforated plate with single or double hole pattern connected to the fixed closed-type container which is made of carbon steel. The plate is made of 321 SS having a radius of 1384 mm and a thickness of 30 mm and 3 mm (Fig. 2). The physical properties of the material at 310℃ are as follows: Young's modulus=173.0 GPa, Poisson's ratio=0.3, and mass density=8027 kg/m$^3$. Water is used as the contained fluid, having a density of 703 kg/m$^3$. The sound speed in water is 653 m/sec, which is an equivalent to the bulk modulus of elasticity of 0.30 GPa.

The frequency equations derived in the preceding sections involve an infinite series of algebraic terms. Before exploring the analytical method to obtain the natural frequencies of the fluid-coupled plate, it is necessary to conduct convergence studies and establish the number of terms required in the series expansions involved. In the numerical calculation, the Bessel-Fourier expansion term $s$ is set to 200 and the expanding term $m$ (or $M$) for the admissible function is set to 40, which gives an exact enough solution by convergence.

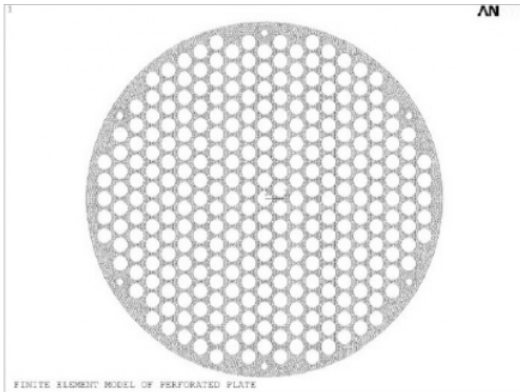
### 3.2 Finite element analysis

Finite element analyses using a commercial computer code ANSYS 8.1 (ANSYS, 2004) are performed to verify the analytical results for the theoretical study. The results from finite element method are used as the baseline data. Three different three-dimensional models are developed for perforated plates with single pattern and double hole pattern, and solid plate as shown in Fig. 3. Also, the solid plate submerged in fluid is modeled, where the fluid region is divided into a number of 3-dimensional contained fluid elements (FLUID80) with eight nodes having three degrees of freedom at each node. The fluid element FLUID80 is particularly well suited for calculating hydrostatic pressures and fluid/solid interactions. The circular plate is modeled as elastic shell elements (SHELL63) with four nodes.

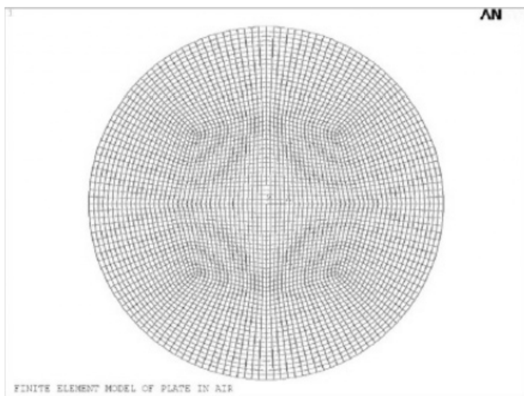
The perimeter nodes of the plate are coupled with the nodes of the container which are fixed in all six degrees of freedom. The fluid movement at top and bottom of the container is considered to be constrained in the vertical direction for the bounded surface fluid case. The vertical velocities




(a) Perforated plate with double hole




(b) Perforated plate with single hole




(c) Solid plate

**Fig. 3** Finite element models

of the fluid element nodes adjacent to each surface of the wetted circular plate coincide to those of plate so that the model can simulate Eqs. (9a) and (9b).

Several cases of the finite element analyses are performed as in Table 2 depending on the temperature, plate thickness or plate modeling etc. The Block Lanczos method is used for the eigenvalue and eigenvector extractions to calculate 300 frequencies including fluid modes (Grimes et al., 1994). It uses the Lanczos algorithm where the Lanczos recursion is performed with a block of vectors. This method is as accurate as the subspace method, but faster. The Block Lanczos method is especially powerful when searching for eigenfrequencies in a given part of the eigenvalue spectrum of a given system. The convergence rate of the eigenfrequencies will be about the same when extracting modes in the midrange and higher end of the spectrum as when extracting the lowest modes.

## 4. Results and Discussion

The frequency comparisons between analytical solution developed here and finite element method are shown in Fig. 4 for plates with thickness of 30 mm in air, respectively. The symbol $m$ in the tables represents the number of nodal circles of the mode and the symbol $n$ means the number of nodal diameter. The frequency differences of plate in air are almost negligible and the comparison of frequencies between thickness of 30 mm and 3 mm shows that the frequencies are proportional to the thickness. This is also shown from the theory as follows.

Introducing Rayleigh quotient of the circular plate in air, Eq. (10) becomes for $T_F=0$ as

$$\omega^2=\frac{V_d}{T_d} \tag{15}$$

where $V_d$ and $T_d$ are determined from Eqs. (14) and (13), respectively. Insertion of Eqs. (14) and (13) into Eq. (15) and the relation between the natural frequency and the thickness of the solid plate give

$$\omega\propto\sqrt{\frac{D}{\rho h}}=\sqrt{\frac{Eh^3}{12\rho h(1-\mu^2)}}=h\sqrt{\frac{E}{12\rho(1-\mu^2)}} \tag{16}$$

Or, the natural frequency of the plate in air is proportional to the thickness of the plate. The relation between the natural frequency and Poisson's ratio is also determined from Eq. (16) and is shown in Fig. 5.

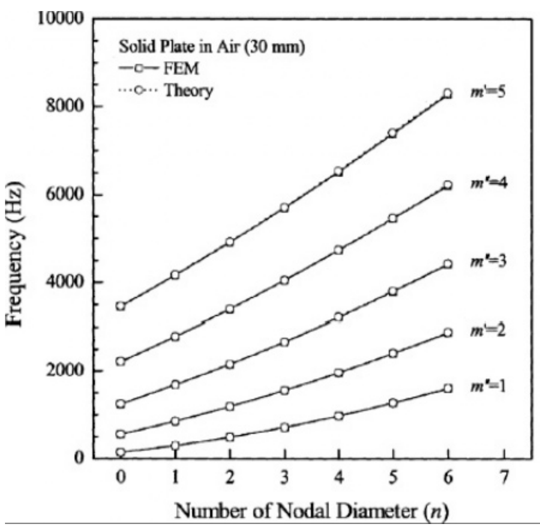



**Fig. 4** Frequency comparisons of solid plate between FEM and theory (thickness=30 mm)

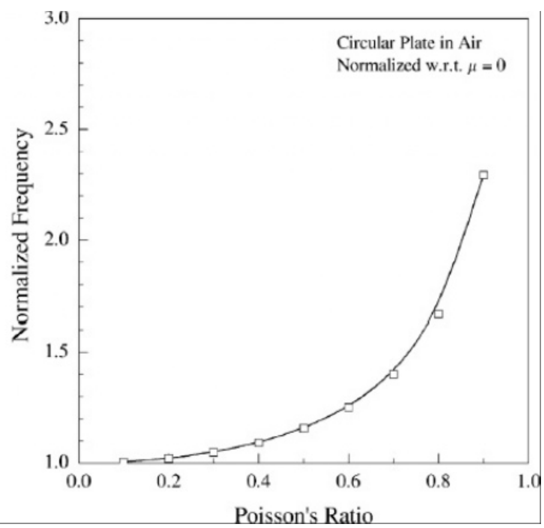



**Fig. 5** Relation between natural frequency and Poisson's ratio

For the plate submerged in fluid, Eq. (10) becomes so complicated and the relation between the natural frequency and the plate thickness is not expressed explicitly as in the case of the in-air condition, which makes it hard to predict modal characteristics of the plate submerged in fluid with respect to the plate thickness.

The frequency comparisons between theory and finite element method are shown in Table 1 for plate submerged in fluid with bounded surface. There is a good agreement between them with the discrepancies of less than 1%.

Frequencies of solid plate and perforated plate with single or double hole pattern and their normalized values are shown in Figs. 6 and 7, respec-

**Table 1** Comparison of natural frequencies between theoretical and FEM results for the solid plate submerged in fluid

| Mode | | 310℃ | | |
|---|---|---|---|---|
| $n$ | $m$ | FEM | Theory | Error (%) |
| 1 | 1 | 66.8 | 66.8 | 0.00 |
| | 2 | 276.6 | 276.7 | −0.04 |
| | 3 | 336.6 | 336.7 | −0.03 |
| | 4 | 746.9 | 747.4 | −0.07 |
| | 5 | 801.1 | 801.1 | 0.00 |
| 2 | 1 | 165.4 | 165.4 | 0.00 |
| | 2 | 458.9 | 458.8 | 0.02 |
| | 3 | 524 | 524.3 | −0.06 |
| | 4 | 968.2 | 969.2 | −0.10 |
| | 5 | 1007.6 | 1007.5 | 0.01 |
| 3 | 1 | 298.8 | 298.9 | −0.03 |
| | 2 | 631.1 | 631.3 | −0.03 |
| | 3 | 726.3 | 727 | −0.10 |
| | 4 | 1200.5 | 1202.3 | −0.15 |
| | 5 | 1204.3 | 1204.3 | 0.00 |
| 4 | 1 | 460.6 | 461 | −0.09 |
| | 2 | 798.8 | 798.9 | −0.01 |
| | 3 | 934.4 | 935.5 | −0.12 |
| | 4 | 1394.7 | 1394.7 | 0.00 |
| | 5 | 1444.4 | 1447.1 | −0.19 |
| 5 | 1 | 643.6 | 644.4 | −0.12 |
| | 2 | 963.8 | 964 | −0.02 |
| | 3 | 1144.5 | 1146.1 | −0.14 |
| | 4 | 1580.6 | 1580.6 | 0.00 |
| | 5 | 1696.3 | 1699.8 | −0.21 |
| 6 | 1 | 840 | 841.2 | −0.14 |
| | 2 | 1126.8 | 1127.1 | −0.03 |
| | 3 | 1356.3 | 1358.5 | −0.16 |
| | 4 | 1763.1 | 1763.2 | −0.01 |
| | 5 | 1949.7 | 1953.7 | −0.21 |

* Error = (FEM − Theory)/FEM × 100

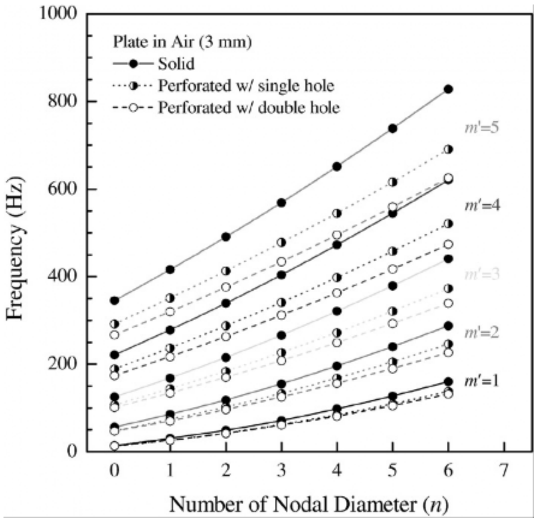



**Fig. 6** Frequency comparisons between solid and perforated plates in air

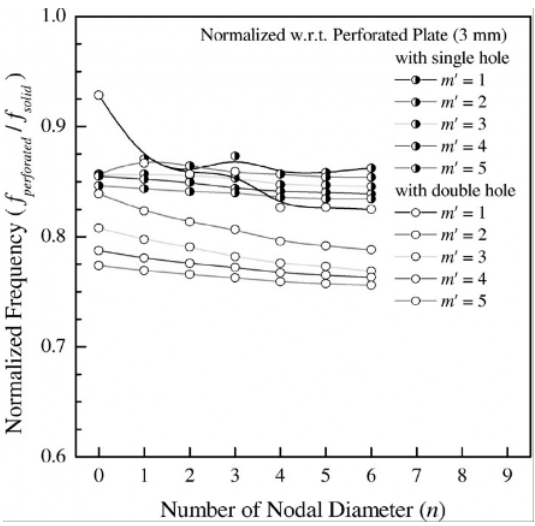



**Fig. 7** Normalized frequencies of perforated plate w.r.t. solid plates in air

tively. In all cases, as the number of nodal circles increases, frequencies increase, which were not shown in cylindrical shells (Jhung et al., 2002 ; Jhung et al., 2003). The inclusion of holes decreases the frequency significantly and the effect of holes is more significant for double hole pattern and this is more significant as the modal number increases. Typical mode shapes of radial

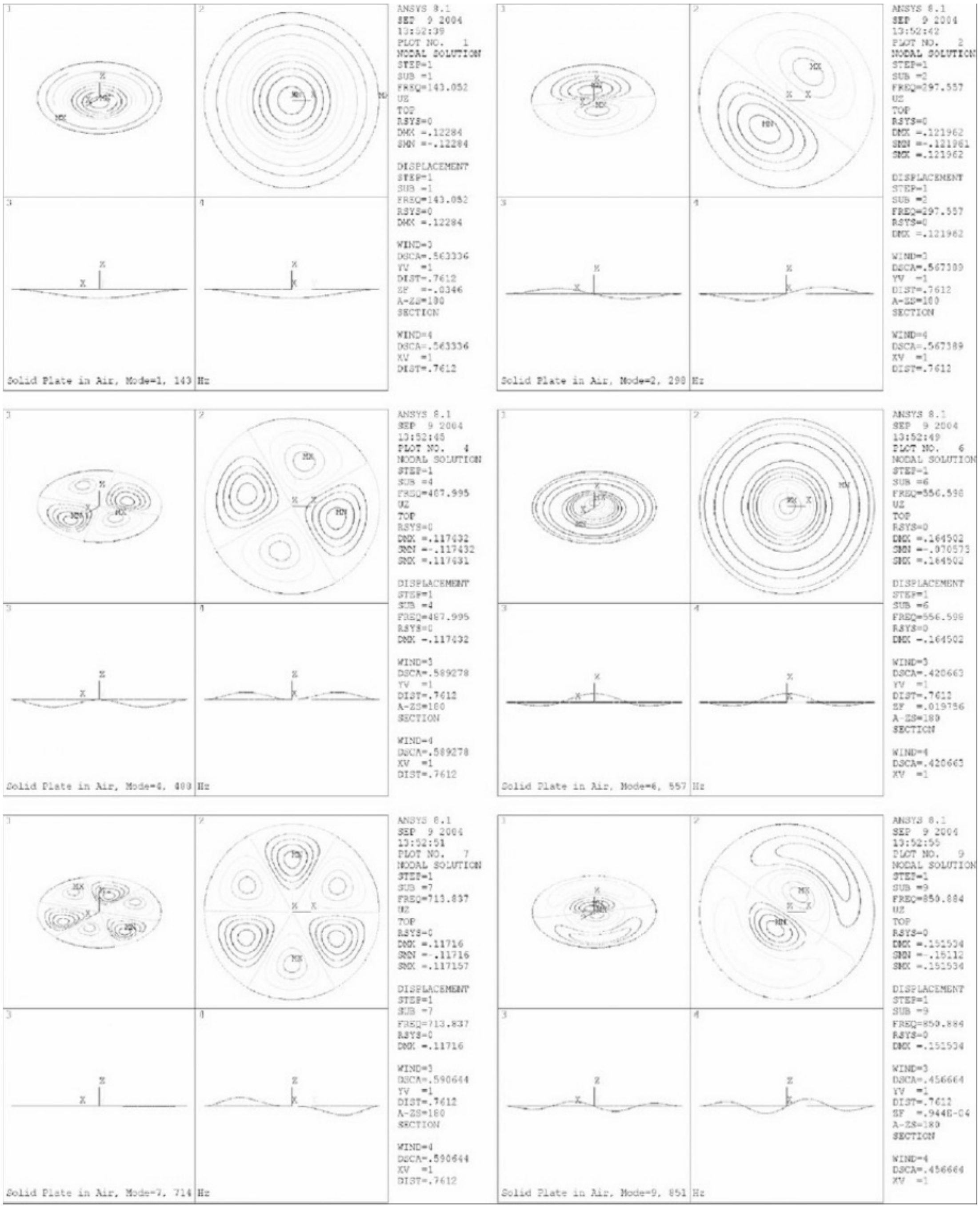



**Fig. 8** Typical mode shapes of solid plate

modes are shown in Figs. 8 and 9 for the solid and perforated plate, respectively.

The frequency comparisons between plate in air and plate submerged in fluid are shown in Fig. 10. The effect of fluid on the frequencies of circular plate wetted with fluid can be assessed using the normalized frequency defined as the natural frequency of a structure in contact with a fluid

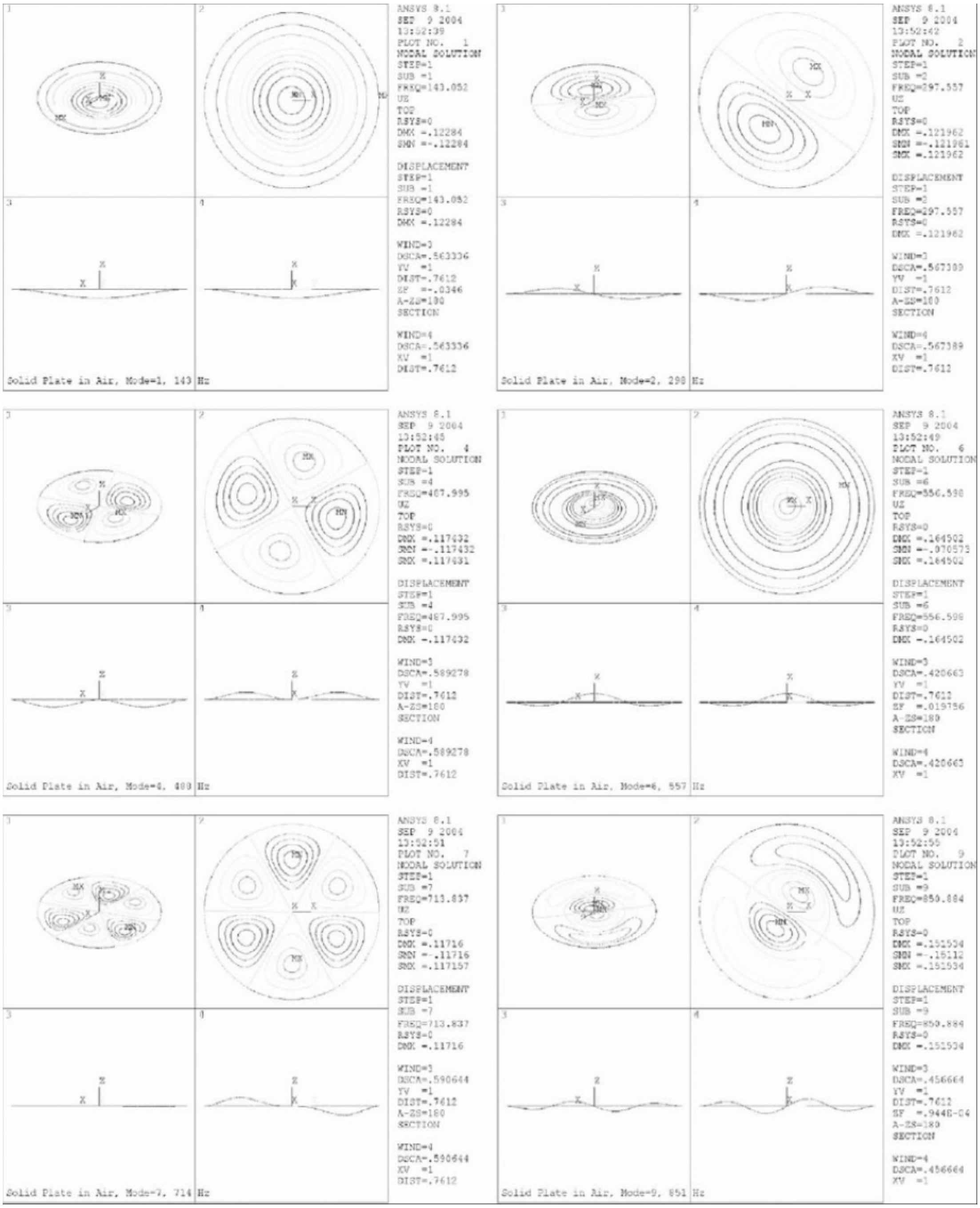



**Fig. 8** Typical mode shapes of solid plate (Cont'd)

divided by the corresponding natural frequency in a vacuum. The normalized natural frequencies have values between one and zero due to the added mass effect of fluid. Fig. 11 shows the normalized natural frequencies for the plate modes submerged in fluid. As the number of nodal circles or diameters of the plates increases, the normalized natural frequencies increase by the grad-

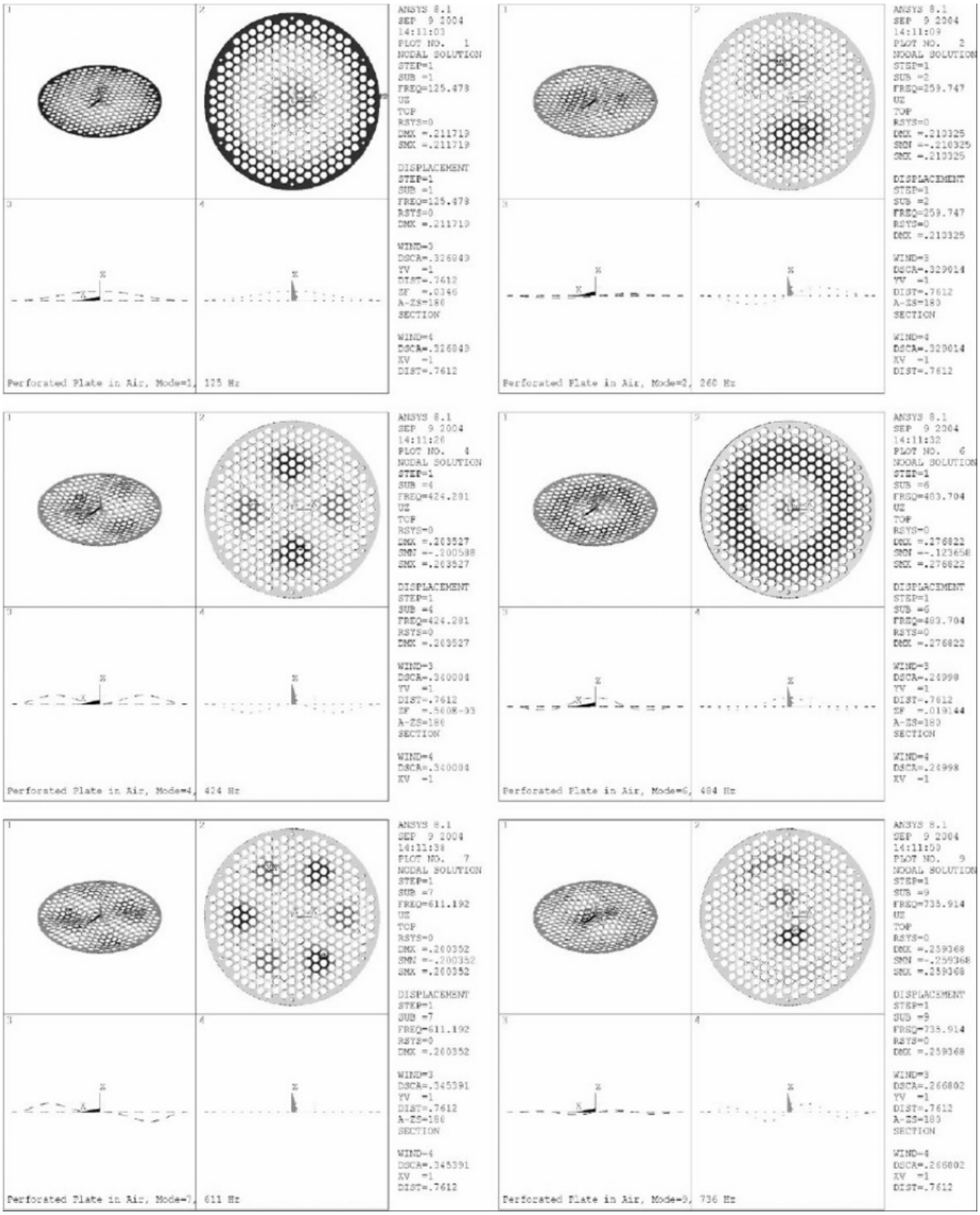



**Fig. 9** Typical mode shapes of perforated plate

ual reduction of the relative added mass effect. Therefore, an increase of nodal lines or nodal circles causes an increase in the normalized natural frequencies for all cases of modes.

The frequency comparisons between perforated plate with original properties and solid plate with equivalent properties in air are shown in Fig. 12, where the equivalent material properties

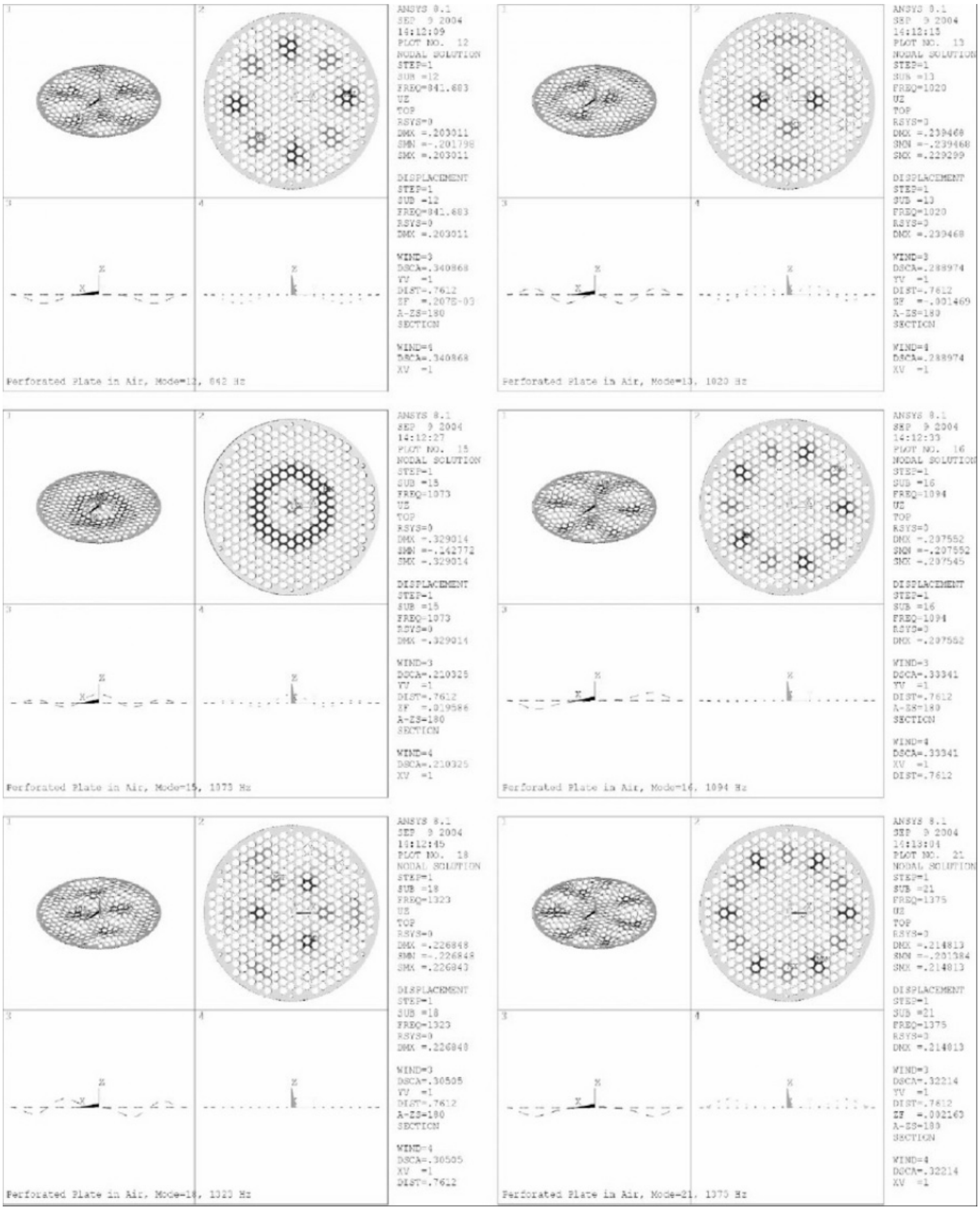



**Fig. 9** Typical mode shapes of perforated plate (Cont'd)

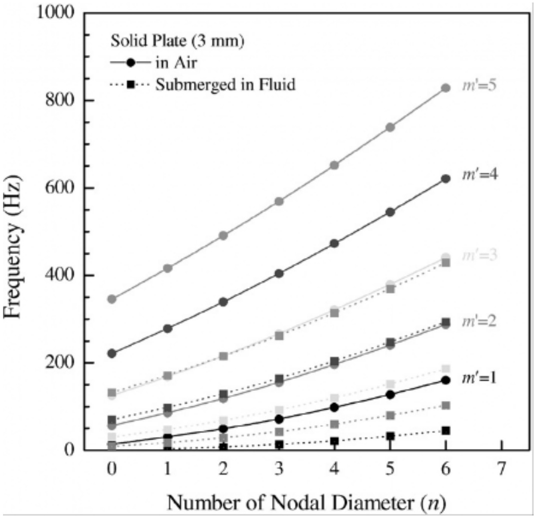



**Fig. 10** Frequency comparisons of solid plate between in air and submerged in fluid

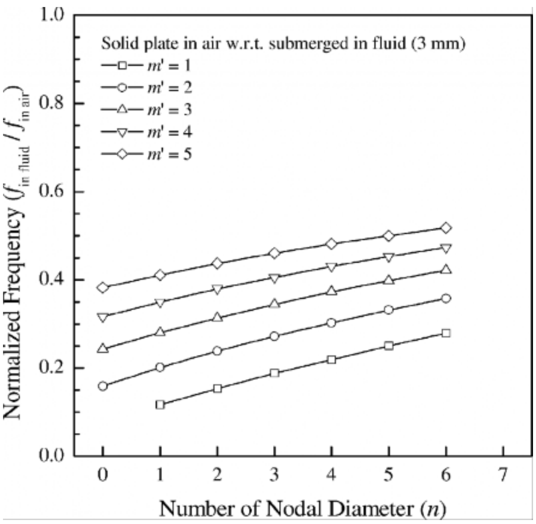



**Fig. 11** Normalized frequencies of solid plate submerged in fluid w.r.t. in air

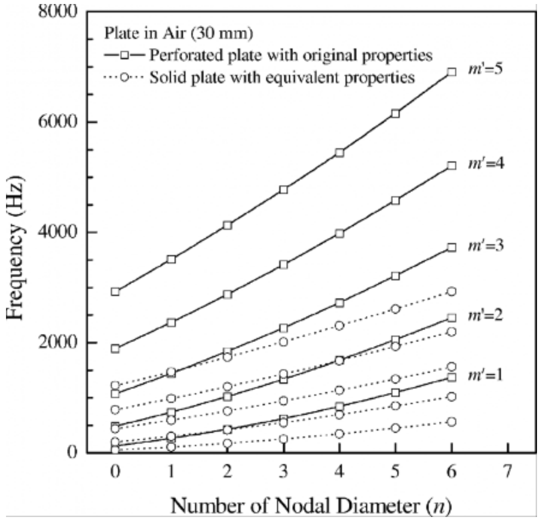



**Fig. 12** Frequency comparisons between perforated plate with original properties and solid plate with equivalent properties in air (thickness = 30 mm)

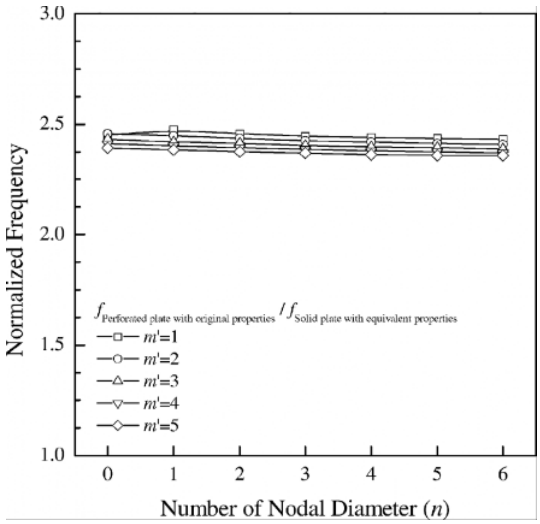



**Fig. 13** Normalized frequencies of perforated plate with original properties with respect to solid plate with equivalent properties in air (thickness = 30 mm)

are calculated from Article A-8000 of Appendix A to the ASME code Section III (ASME, 2004). Their normalized values shown in Fig. 13 indicates that using effective elastic constants of ASME code generates too low frequencies and therefore it is not proper to use the solid plate model with the effective elastic constants provided by the ASME code for the modal analysis of the perforated plate. Because the frequency is proportional to the square root of the elastic constant, the effective elastic constants must be increased by the ratio of about 6 to match the frequencies. The effective elastic constants given in the ASME code are restricted to plates having a thickness $t$ of greater than twice the pitch of the hole pattern $P$ and the model studied here has $t/P = 30/72 =$

0.42 which is not proper to apply the ASME code values. But the effective elastic constants generated by O'Donnell (1973) for the thin perforated plate are also found to be too low. This necessitates the redefinition of the equivalent material properties for the modal characteristics of the perforated plate.

Considering a circular perforated plate with a triangular penetration pattern as shown in Fig. 14, where the radius of the plate and pitch of the hole are 1384 mm and 72 mm, respectively, the modal characteristics and the equivalent elastic constants are investigated as a typical case.

As mentioned earlier, the equivalent elastic constants of perforated plate with a triangular penetration pattern proposed by the ASME code are found to be no more valid for the modal analysis. Therefore it is necessary to redefine the equivalent elastic constants such that the modal characteristics of the perforated plate with original properties be the same with those of the solid plate with modified equivalent properties. The best way to find the equivalent constant is as follows :

(a) Develop finite element model of solid plate and perforated plates with various ligament efficiencies.

(b) Perform the modal analyses of perforated and solid plate with original properties.

(c) Compare the frequencies and find the ratio of frequencies of perforated plate to those of solid plate.

(d) Find the multipliers of Young's modulus of solid plate to match frequencies of perforated plate with original properties using the relations between frequency and Young's modulus.

(e) Effective elastic constant for each ligament efficiency is averaged for all modes.

This kind of analysis is repeated for several ligament efficiencies $\eta = h/P$. Because the frequency is proportional to the thickness of the plate even if it is perforated, it is not necessary to perform several analyses with respect to the thickness. Therefore in this study the thickness ratio of $t/P = 0.05$ is considered for the typical case.

The natural frequencies are summarized in Fig. 15 and their normalized values of perforated plate with respect to the solid plate are obtained. Because the elastic constant is proportional to the square of the frequency, the normalized frequencies are squared and these are the effective elastic constants with which the same modal frequencies as the perforated plate are obtained for the solid

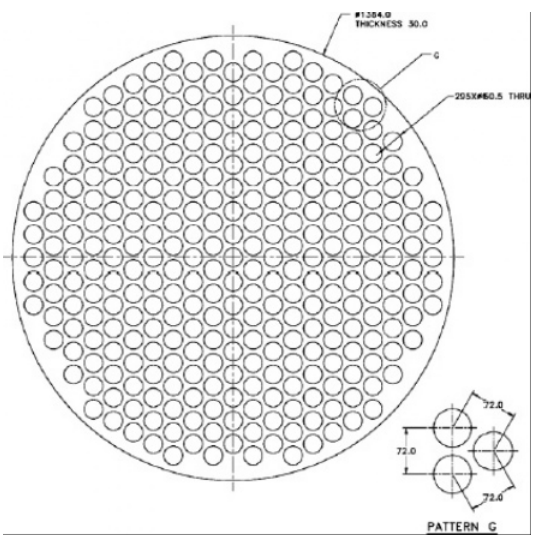



**Fig. 14** Perforated plate with a triangular penetration pattern

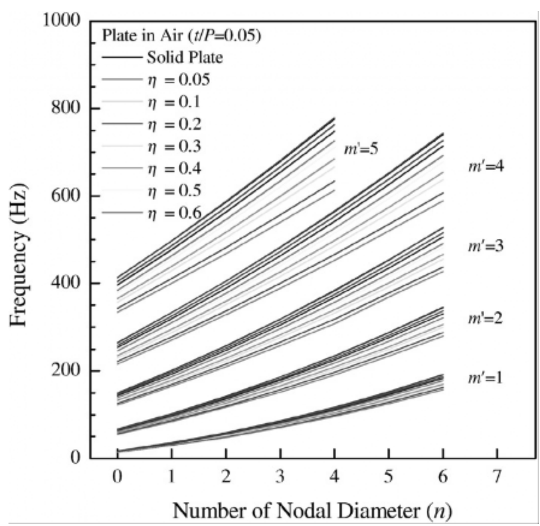



**Fig. 15** Natural frequencies of plate in air for solid plate and perforated plates with various ligament efficiencies

plate. Because the effective values are depend on the mode numbers, their average values are used. There is a large difference between effective elastic constants of Slot and O'Donnell (1971) and those determined here. Even though values from Slot and O'Donnell are for the thick plate $t/P \geq 2.0$, it is not assumed to be valid from the modal characteristic point of view for the perforated plate due to the fact that the frequency is proportional to the thickness of the plate. Figure 16 is the final effective elastic constants proposed for modal characteristics of the perforated plate with a triangular penetration pattern, which can be represented for $0.05 \leq \eta \leq 0.8$ as:

$$\frac{E^*}{E} = 0.6106 + 1.1253\eta - 2.7118\eta^2 + 4.0812\eta^3 - 2.1128\eta^4 \quad (17)$$

The natural frequency of the plate is proportional to the thickness of the plate even for the thick one, which shows the frequencies of the perforated plate with respect to the thickness with a ligament efficiency of 0.05. Therefore effective elastic constants proposed in Fig. 16 can be used for all thickness range of the plate.

Figure 17 shows the frequency comparisons between perforated plate with original properties and solid plate with effective elastic constant determined in this study and it is found to be in good agreement between them with the difference of less than 3% verifying the validity of the method developed here to calculate the effective elastic constants for the modal analysis of the perforated plate with a triangular penetration pattern.

Using the effective Young's modulus redefined, the frequencies of perforated plate with triangular penetration pattern submerged in fluid are

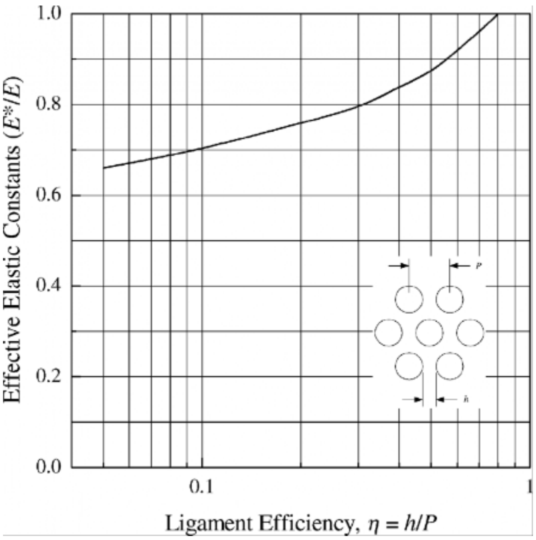



**Fig. 16** Effective elastic constants of perforated plate with a triangular penetration pattern

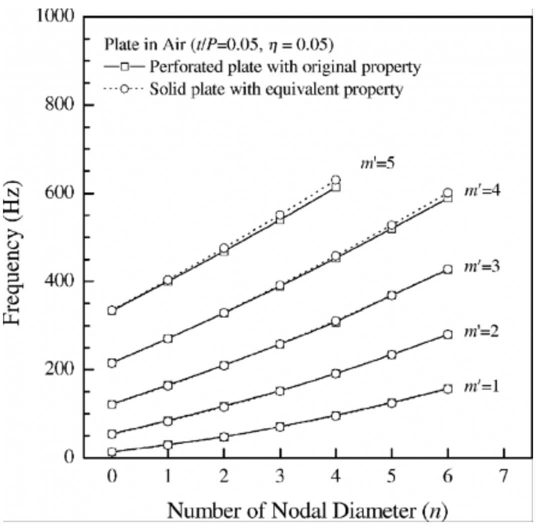



**Fig. 17** Comparison of frequencies between perforated plate with original property and solid plate with equivalent property

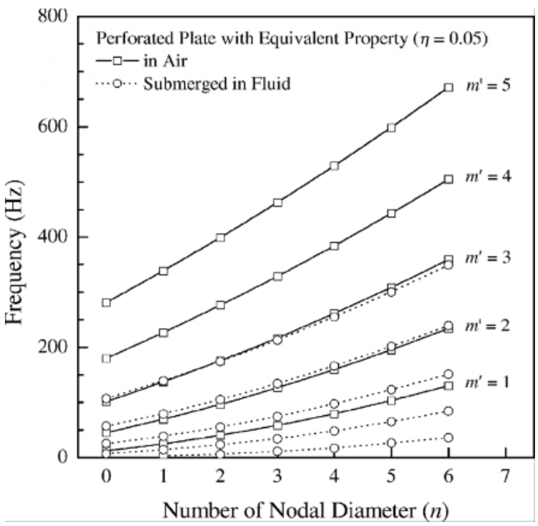



**Fig. 18** Frequency comparisons of perforated plate between in air and submerged in fluid

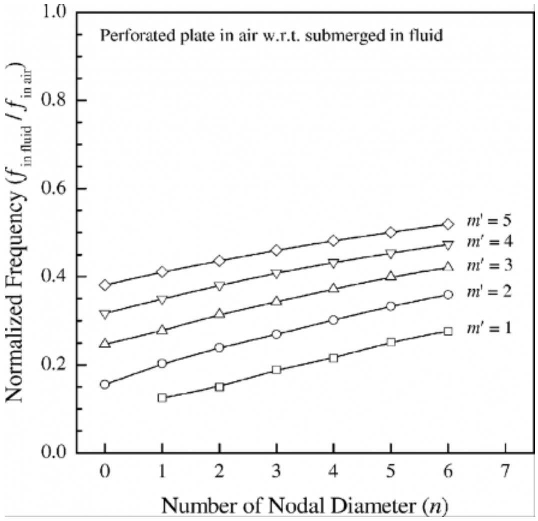



**Fig. 19** Normalized frequencies of perforated plate submerged in fluid w.r.t. in air

calculated assuming that no cross flow of fluid occurs. The equivalent Young's modulus for $\eta=0.05$ is $E^*/E=0.66$. The frequencies for the equivalent Young's modulus are shown in Fig. 18 for perforated plates in air and submerged in fluid and normalized frequencies are also shown in Fig. 19. They are comparable to Figs. 11 and 12 which show the frequency comparisons of solid plate between in air and submerged in fluid and their normalized values. By comparing normalized frequencies between Fig. 11 and Fig. 19, the ratio of frequency reduction due to the added mass effect of fluid is found to be the same for the solid plate and perforated plate.

## 5. Conclusions

An analytical method to estimate the coupled frequencies of the circular plate submerged in fluid is developed using the finite Fourier-Bessel series expansion and Rayleigh-Ritz method. To verify the validity of the analytical method developed, finite element method is used and the frequency comparisons between them are found to be in good agreement.

For the perforated plate submerged in fluid, it is almost impossible to develop a finite element model due to the necessity of the fine meshing of the plate and the fluid at the same time. This necessitates the use of solid plate with equivalent material properties. Unfortunately the effective elastic constants suggested by the ASME code are found to be not valid for the modal analysis. Therefore in this study the equivalent material properties of perforated plate are suggested by performing several finite element analyses with respect to the ligament efficiencies.

Using the equivalent Young's modulus defined in this study, the modal analysis of the perforated plate submerged in fluid is performed. The normalized frequencies of perforated plate submerged in fluid with respect to the perforated plate in air are calculated and they are compared with those of solid plate. It is found that the frequency reduction due to the added mass effect of the fluid is the same irrespective to the plate whether it is solid or perforated.

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